Teacher notes
Topic E

Helium and the Sun.

(a) The solar constant is 1360 W m^{-2} and the earth-Sun distance is 1.5×10^{11} m. Calculate the total energy produced in the Sun during its lifetime so far of 4.5 billion years.

(b) The energy of the Sun comes from nuclear fusion reactions in which 4 hydrogen nuclei fuse into one helium nucleus: $4\times {}^{1}_{1}\mathrm{H}\rightarrow {}^{4}_{2}\mathrm{He}+2e^{+}+2\nu+2\gamma$.

Nuclear mass of ${}^{1}_{1}\mathrm{H}$ 1.007276 u
Nuclear mass of ${}^{4}_{2}\mathrm{He}$ 4.001505 u

(i) Show that the energy released is 26.7 MeV. You must include 2.04 MeV produced when the two positrons annihilate with two electrons.
(ii) Estimate the mass of helium produced in 4.5 billion years as a percentage of the solar mass of 2.0×10^{30} kg.

(c) The observed mass of helium now as a percentage of the total mass of the Sun is 25%. Comment on the answer to (b)(ii) considering this fact.

(d) For each helium atom produced, the two neutrinos have an average energy of 0.3 MeV each. Estimate

(i) the number of neutrinos leaving the Sun per second,
(ii) the number of solar neutrinos entering your body (of surface area 0.5 m^2) per second,
(iii) the luminosity of the Sun in the form of neutrinos.

Answers

(a) $L_{\odot} = S \times 4\pi d^2$ and
$E = L_{\odot} t = S \times 4\pi d^2 \times t = 1360 \times 4\pi \times \left(1.5 \times 10^{11}\right)^2 \times \left(4.5 \times 10^9 \times 365 \times 24 \times 3600\right) = 5.5 \times 10^{43}\ \text{J}$.

(b)

(i) $4\times(1.007276) - (4.001505 - 2\times0.000549) = 0.026510$ u so $Q = 0.026510 \times 931.5 = 24.685682$ MeV. Adding 2×1.02 MeV gives 26.73 MeV.

(ii) The total number of fusion reactions is $\frac{5.5 \times 10^{43}}{26.7 \times 10^6 \times 1.6 \times 10^{-19}} = 1.29 \times 10^{55}$. In each reaction one helium nucleus is formed so the mass of helium is $1.29 \times 10^{55} \times 4u = 1.29 \times 10^{55} \times 4 \times 1.66 \times 10^{-27} = 8.6 \times 10^{28}\ \text{kg}$. The mass of the Sun is $2.0 \times 10^{30}\ \text{kg}$ so the helium mass is a percentage of $\frac{8.6 \times 10^{28}}{2.0 \times 10^{30}} \times 100\% = 4.3\%$ of the total solar mass.

(c) This is much smaller than the observed 25%. It means that not all the helium in the Sun was formed through fusion of hydrogen inside the Sun. Some helium was present in the material from which the Sun was made. (The Big Bang model (BB) predicts that helium was formed at the BB so this is evidence for the BB model of the creation of the Universe.)

(d)

(i) The number of reactions per second is $\frac{1.29 \times 10^{55}}{\left(4.5 \times 10^9 \times 365 \times 24 \times 3600\right)} = 9.1 \times 10^{37}$ and in each reaction 2 neutrinos are produced so the total number is $2 \times 9.1 \times 10^{37} = 1.8 \times 10^{38}\ \text{s}^{-1}$.

(ii) The number of neutrinos per unit area per second arriving at earth is $\frac{1.8 \times 10^{38}}{4\pi\left(1.5 \times 10^{11}\right)^2} = 6.4 \times 10^{14}\ \text{m}^{-2}\ \text{s}^{-1}$ and so the number entering your body per second is $0.5 \times 6.4 \times 10^{14} = 3.2 \times 10^{14}\ \text{s}^{-1}$. (If all these neutrinos were absorbed by your body, a power of $0.3 \times 10^6 \times 1.6 \times 10^{-19} \times 3.2 \times 10^{14} = 15\ \text{W}$ would be deposited in the body, which is not insignificant. However, the neutrinos are so weakly interacting that not a single neutrino would be absorbed by the body in your lifetime!)

(iii) The total energy carried away per second by the neutrinos (the neutrino luminosity) is then $0.3 \times 10^6 \times 1.6 \times 10^{-19} \times 1.8 \times 10^{38} = 8.6 \times 10^{24}\ \text{J s}^{-1}$. This is a fraction of $\frac{8.6 \times 10^{24}}{3.9 \times 10^{26} + 8.6 \times 10^{24}} = 0.022$ or 2.2% of the total solar luminosity.